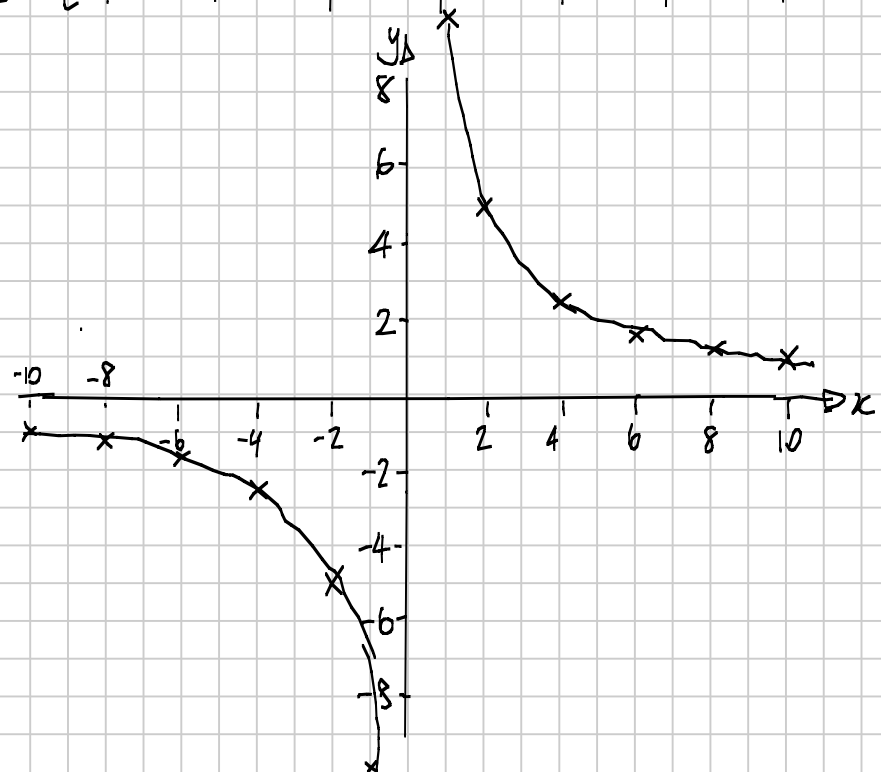
C4 Exercise 2A (parametric equations intro.)

Note Title 12/11/2010

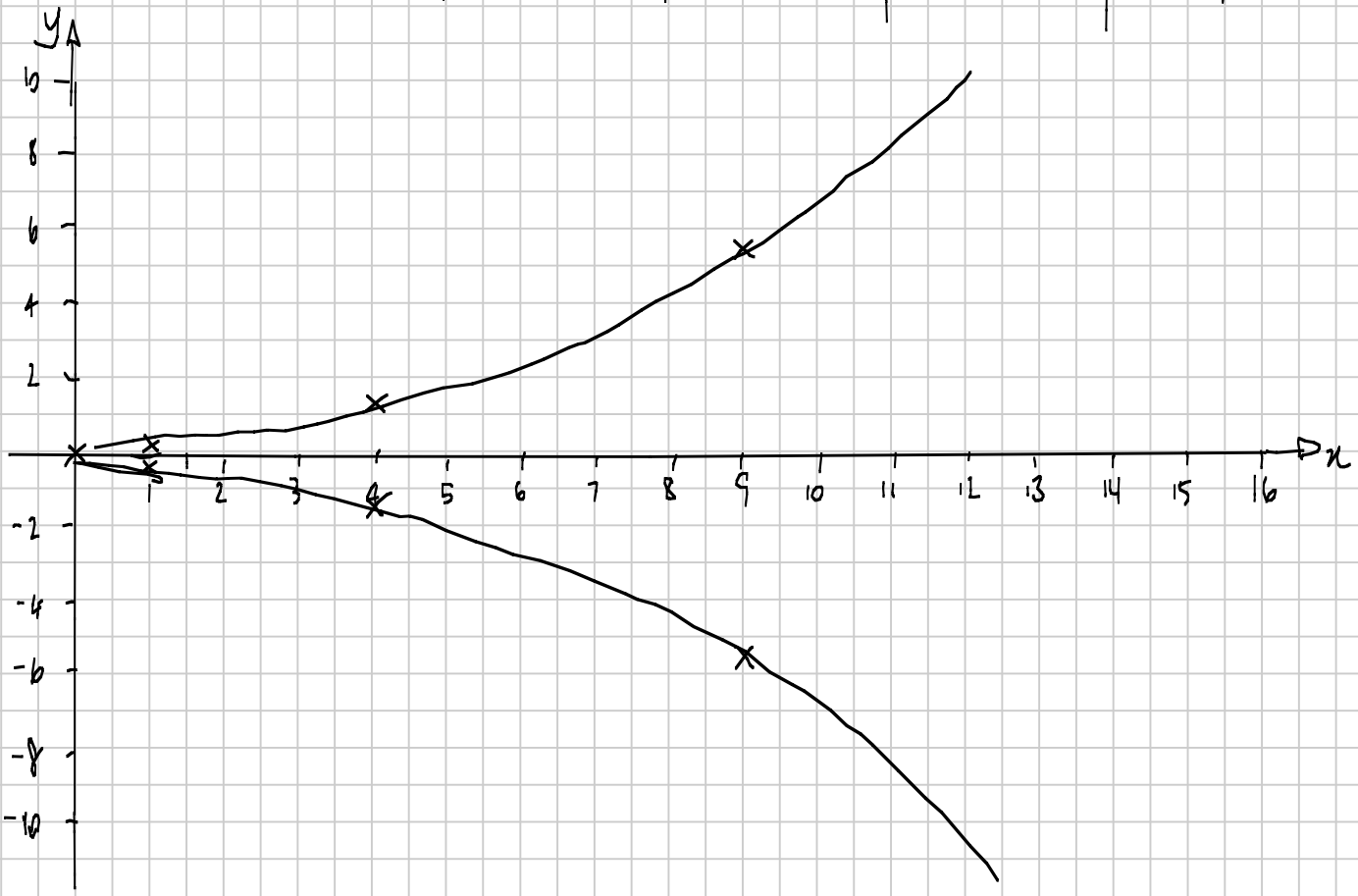
1 A curve is given by the parametric equations $x = 2t$ and $y = \frac{5}{t}$ where $t \neq 0$. Complete the table & plot the graph for $-5 \leq t \leq 5$.

t	-5	-4	-3	-2	-1	-0.5	0.5	1	2	3	4	5
$x=2t$	-10	-8	-6	-4	-2	-1	1	2	4	6	8	10
$y=\frac{5}{t}$	-1	-1.25	$-1.\dot{6}$	-2.5	-5	-10	10	5	2.5	$1.\dot{6}$	1.25	1



2 A curve is given by the parametric equations $x = t^2$ and $y = \frac{t^3}{5}$. Complete the table & plot the graph... for $-4 \leq t \leq 4$.

t	-4	-3	-2	-1	0	1	2	3	4
x	16	9	4	1	0	1	4	9	16
y	-12.8	-5.4	-1.6	-0.2	0	0.2	1.6	5.4	12.8



3a Sketch $x = t - 2$ $y = t^2 + 1$ for $-4 \leq t \leq 4$

$t =$	$x = t-2$	$y = t^2+1$
-4	-6	17
-3	-5	10
-2	-4	5
-1	-3	2
0	-2	1
1	-1	2
2	0	5
3	1	10
4	2	17

3c

3d

3e

4a

4c

4e

4j Find the Cartesian equation of the curve given by

$$x = \frac{t}{2t-1}, \quad y = \frac{t}{t+1}, \quad t \neq -1, \ t \neq \frac{1}{2}$$

$$x = \frac{t}{2t-1} \Rightarrow 2tx - x = t$$

multiply to remove fraction

$$\Rightarrow 2tx - t = x$$

move t to LHS non t RHS

$$\Rightarrow (2x-1)t = x$$

$$\Rightarrow t = \frac{x}{2x-1}$$

So $y = \dfrac{\left(\frac{x}{2x-1}\right)}{\left(\frac{x}{2x-1}\right)+1} \Rightarrow y = \left(\frac{x}{2x-1}\right) \div \left(\frac{x+(2x-1)}{2x-1}\right)$

$$\Rightarrow y = \left(\frac{x}{2x-1}\right) \times \left(\frac{2x-1}{3x-1}\right) \Rightarrow y = \frac{x}{3x-1}$$

5 Show that the parametric equations

(i) $x = 1 + 2t$, $y = 2 + 3t$

and (ii) $x = \dfrac{1}{2t-3}$, $y = \dfrac{t}{2t-3}$, $t \neq 3/2$

represent the same straight line

(i) rearranges to give $x = 1 + 2t$

$$\Rightarrow t = \frac{x-1}{2}$$

hence $$y = 2 + 3\left(\frac{x-1}{2}\right)$$

$$\Rightarrow y = \frac{3}{2}x + \frac{1}{2}$$

This is clearly a straight line.

(ii) rearranges to give $x = \dfrac{1}{2t-3}$

$\Rightarrow \quad 2tx - 3x = 1$

$\Rightarrow \quad 2tx = 3x + 1$

$\Rightarrow \quad t = \dfrac{3x+1}{2x}$

hence $y = \dfrac{\left(\frac{3x+1}{2x}\right)}{2\left(\frac{3x+1}{2x}\right) - 3}$

$\Rightarrow \quad y = \left(\dfrac{3x+1}{2x}\right) \div \left(\dfrac{\cancel{3x}+1}{x} - \dfrac{\cancel{3x}}{x}\right)$

$\Rightarrow \quad y = \left(\dfrac{3x+1}{2x}\right) \div \left(\dfrac{1}{x}\right)$

$$\Rightarrow \quad y = \left(\frac{3x+1}{2x}\right) \times x$$

$$\Rightarrow \quad y = \frac{3}{2}x + \frac{1}{2}$$

And this is the same straight line as in (i).